Journal of Engineering Physics and Thermophysics, Vol. 75, No. 2, 2002

HYDRODYNAMICS OF ROTORY-PULSATORY APPARATUSES

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UDC 532.695

A kinematic and dynamic analysis of flows in a rotory-pulsatory apparatus has been carried out. Based on this analysis, a method for calculating the main parameters of the apparatus with the use of minimum initial experimental information (the head at zero transit flow rate H_0 and the flow rate at zero head Q_0) has been developed.

Rotory-pulsatory (RPA) or rotory-impulse apparatuses (RIA) have found wide application in chemical, pharmaceutical, and food engineering for production of finely dispersed emulsions and suspensions. They represent structures from coaxial cylindrical shells with openings (slots, windows), the even (or odd) of which are set in rotation. If such a system is positioned in a liquid medium, the rotating shells will initiate the azimuthal circular motion of a liquid in the intercylinder gaps and, under the action of centrifugal forces, the radial movement of the medium.

A number of investigations are devoted to the analysis of the operation of such apparatuses [1–7]. However, we cannot agree with all the theoretical premises. For example, in [1] it is assumed without proper substantiation that the conditions of motion in the intercylinder gap are turbulent and the calculation is made with the use of a certain three-layer model of flow: a viscous flow in the near-wall regions of the intercylinder gap and a turbulent core between them. The radial movement of the liquid is completely ignored. In [2], the analysis of the flow in the window of an immovable cylinder is based on the jet scheme, according to which the oppositely directed motions of the liquid must occur along the outer boundaries of this cylinder, with which we cannot agree. Moreover, the momentum equation does not take into account the contribution of the radial component of the pressure gradient. The head flow in intercylinder gaps, which, as a rule, is not realized in RPA, is the focus of [6]. In [3], information on the hydrodynamics is practically absent.

A structure from three coaxial cylindrical shells, the central of which is set in rotation, can be considered as the base element of RPA. The schematic diagram of the simplest RPA is given in Fig. 1. The main technological function of an RPA is to attain a high homogenizing effect. Because of this, the intercylinder gaps are small ($\delta \sim 0.15$ mm), which determines a rate of shear in them of the order of 10^5 sec^{-1} or more. However, when the width of the gaps δ decreases, there arises a tendency toward increasing the radial flow of a liquid that bypasses the intercylinder gaps and, consequently, does not undergo homogenization. Indeed, even in the case of very low velocities of the radial flow u_r as compared to the velocity of the circular flow u_φ because of the significantly larger mean size of the windows $\langle a \rangle$ as compared to the width of the gap δ the radial flow rate of a nonhomogenized liquid Q_r and the total transit flow rate Q in an RPA can be comparable. Of course, there are technical means that make it possible to decrease $\langle a \rangle$ and even provide $Q_r \to 0$. However, the output of the apparatus decreases significantly in this case. In this connection, it is important to determine the relations between the above-mentioned flow rates for each RPA, which can be done only by way of calculation based on the analysis of the kinematics and dynamics of the flows in the RPA. This is one of the main concerns of this work.

Thus, the following flows and the corresponding flow rates must be distinguished in an RPA:

(a) the circular (azimuthal) flow along closed circular trajectories with flow rate Q_ω;

(b) the circular (azimuthal) transit flow (through the RPA) with flow rate Q_φ;

(c) the radial transit flow bypassing the gaps with flow rate Q_r;

(d) the resultant transit flow through the RPA with flow rate $Q = Q_\varphi + Q_r$, which corresponds to the volumetric output of the RPA.

Institute of Technical Thermophysics, National Academy of Sciences of Ukraine, Kiev, Ukraine. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 75, No. 2, pp. 58–68, March–April, 2002. Original article submitted February 13, 2001; revision submitted July 5, 2001.

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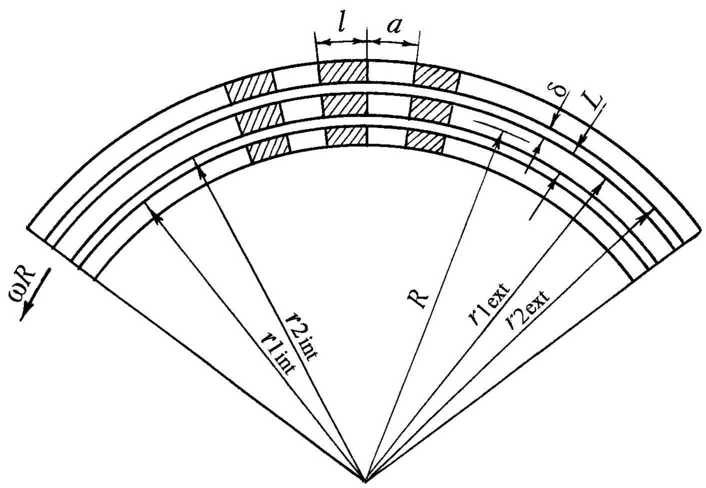


Fig. 1. Schematic diagram of an RPA.

Let us consider the kinematics of the circular transit flow. Particles entrained into the interior intercylinder slot are accelerated in it to the rotor speed and only after that can they enter the window of the rotor and move in it to the exterior intercylinder slot. Particles can enter the window of the exterior stator only after their deceleration in the exterior ring gap of the rotor. Because of this, the particles of the circular transit flow must traverse in their motion at least two regions of the intercylinder gaps, as shown in diagram 1 in Fig. 2, which reflects the moment when the openings in all three cylinders are coincident. When the central cylinder rotates in the direction indicated by the arrow, the condition of coincidence of the openings sets in after a time when the elements of the walls of the moving cylinder a, b, and c, the successive change in the positions of which is shown in diagrams 2, 3, and 4 in Fig. 2, occupy the position shown in diagram 5 in Fig. 2. Diagram 5 is not fundamentally different from diagram 1. Because of this the lines of the particle flow shown in diagram 5 must be identical to those in diagram 1. However, they turn out to be *noncoincident* relative to the elements of the moving wall. Whereas in diagram 1 the line of the particle flow from the window B (solid line) passed through the intercylinder gap *internal* relative to the element b, in diagram 5 this line of flow passes exclusively through the intercylinder gap *external* relative to the element b. It is beyond reason to suggest a jump-like change in the positions of the lines of flow. It would appear reasonable that there is a two-sided flow of particles arriving from the window B around the element b with a certain coefficient of distribution of the flow rate k proportional to the number of particles flowing over the exterior of the element, which is shown in diagrams 2, 3, and 4 in Fig. 2. In this case, the coefficient k changes from zero in scheme 1 to unity in scheme 5 ($k \in [0, 1]$). It seems plausible that k changes linearly with time, since as the time of contact of the particles with the element b increases, their velocity approaches the velocity of the element b, which is favorable to the entry of the particles into the opening of the moving cylinder between the elements b and c. When the number of cylindrical shells in the RPA and the order of their rotation change, it is necessary to refine the kinematic scheme of the circular transit flow.

As for the regime of circular flow, its stability should be analyzed with the use of the calculation scheme for the theoretically and experimentally evaluated case of an RPA in which the liquid is moving in the gap between coaxial cylinders, one of which rotates. In this case, according to Fig. 1, the situation where both the exterior cylinder and the interior shell rotate about the exterior immovable cylinder is realized in the RPA. The difference of the RPA from the above-described cases is that there are openings in its cylinders and, consequently, a distinctly *undirectional* radial flow is realized in it. As is known, radial flow is the main factor disturbing the flow in the gap between the coaxial cylinders without openings, mainly because of the impermeability of the walls, since the compulsory radial motion caused by centrifugal forces must form a closed circuit in this case, i.e., a vortex superimposed on the main circular flow. In the RPA, the destabilizing influence of such vortices is significantly diminished because of the fairly high "porosity" of the cylinders and, consequently, the realization of a uniquely directed radial flow rate. Because of this, the stability conditions for a flow in circuits with impermeable walls can be extended with a certain "margin" to the case of flow in an RPA.

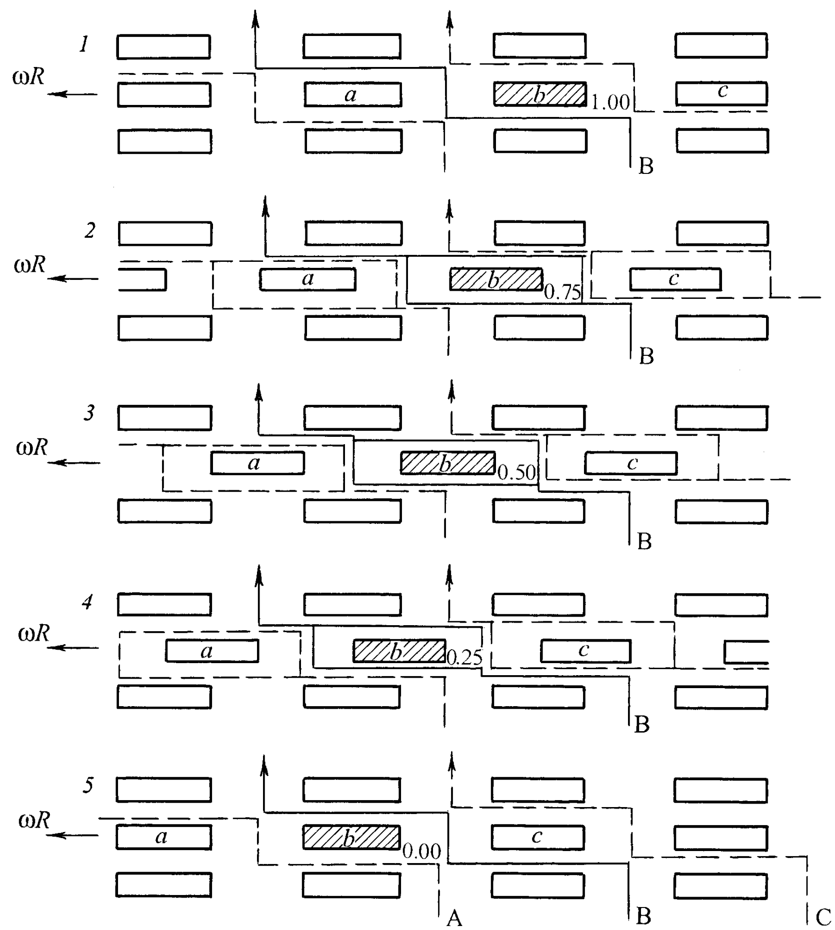


Fig. 2. Kinematics of a circular transit flow. Figures correspond to the values $(1 - k)$.

The analysis of the stability of flow in the intercylinder gap dates back to the fundamental work of Taylor [8], according to which the motion is always stable if the exterior cylinder rotates. Since this method takes no account of viscosity, an additional condition for a purely viscous regime of motion is determined from the lower critical angular velocity ω_* calculated from the empirical Reynolds relation:

$$\frac{\omega_* r_2^2}{\nu}\left(1 - \frac{r_1}{r_2}\right) = 1900 .$$

In the exterior gap, the conditions of preservation of stability for a purely circular motion are less favorable. According to the investigations carried out by Taylor, the stability will be lost when the quantity

$$\mathrm{Ta} = \mathrm{Re}_{\omega_1} \frac{\sqrt{\delta / r_1}}{\bar{F}}$$

exceeds its critical value

$$\mathrm{Ta}_* = \sqrt{48.7\left(2 + \frac{\delta}{r_1}\right)} .$$

Here, the parameters entering into the Taylor dependences are determined by the relations

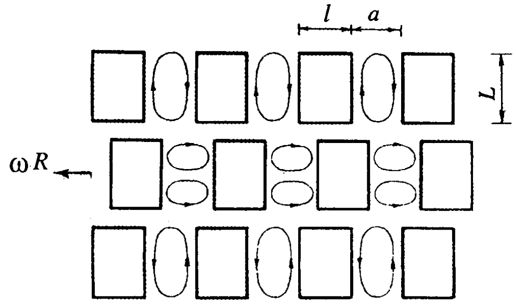


Fig. 3. Vortex system in an RPA.

$$\mathrm{Re}_{\omega_1} = \frac{r_1 \omega_1 \delta}{\nu}, \quad \bar{F} = \frac{\pi^2}{41.2B\sqrt{E}}, \quad B = 1 - \frac{\delta}{2r_1},$$

$$E = 0.0571A - \frac{0.00056}{A}, \quad A = 1 - 0.652C, \quad C = \frac{\delta}{r_1 B}.$$

The above-described kinematic scheme of distribution of circular flows in an RPA must be supplemented with the kinematics of the radial transit flow bypassing the gaps. The rotating interior cylinder represents a diaphragm that is periodically open and closed for the radial flow. Because of this, from the kinematic standpoint, this is a flow through a slot with a variable geometry. It is characterized by a sharp deformation of the flow section and a great nonuniformity of the velocity distribution over the section, which increases because of the intense vortex formation caused by the rotation of the central cylinder with a circular velocity of the order of 10 m/sec or more. In the absence of the transit flow rate, these vortices will correspond to the vortices shown in Fig. 3. Thus, the transit flow in an RPA is the "pumping" of a liquid through the system of high-intensity vortices. Since the kinetic parameters of the transit flow rate, calculated from the formulas for the entire flow section, are small as a rule (two to three decimal orders of magnitude smaller than the parameters of vortex motion), the "pumping" is possible only in the case of concentration of the flow in the form of tubes of flow with a small cross section; these tubes ensure a velocity of motion which makes it possible to pass through the system of vortices. One can assure oneself that this peculiarity of passage of the flow through a high-intensity vortex is true, observing the final stage of emptying of the bath where a particle can pass through a vortex without being entrained into the outflow and only moving with a velocity higher than the local velocity of the vortex particles or moving along the trajectory far removed from the center of the vortex. Figure 4 shows the main lines of flow of circular and radial transit flows. It is easy to calculate the number of rotations of each of the flows in the RPA by the angle $\pi/2$. These are 8 rotations for the first flow and 12 rotations for the second flow.

The vortex formation in the RPA causes a local decrease in the pressure at the centers of the vortices. If a vortex is considered as rectilinear, circular, and cylindrical (induced in a nonviscous medium) the relation between its kinematic and dynamic characteristics is determined in a cylindrical coordinate system by the dynamic equation

$$r\omega^2 = \frac{1}{\rho}\frac{dp}{dr},$$

where ω = const. Integrating this equation with respect to r and denoting the pressure at the boundary of the circular vortex ($r = r_{\mathrm{bound}}$) by p_{bound}, we obtain the expression for the rough value of the pressure at the center of the vortex:

$$p_0 = p_{\mathrm{bound}} - \rho\frac{r_{\mathrm{bound}}^2 \omega^2}{2}.$$

In the prevalent majority of cases, the motion of a mixture in the gaps of the RPA is viscous in character and the rheological equation for the motion of a Newtonian fluid in the gap between the coaxial cylinders has the form

$$p_{r\varphi} = \mu\left(\frac{1}{r}\frac{\partial u_r}{\partial \varphi} + \frac{\partial u_\varphi}{\partial r} - \frac{u_\varphi}{r}\right).$$

It is usual to take $u_r = 0$. Numerous solutions of dynamical problems on the basis of this equation for the cases of rotation of the external cylinder or (and) the internal cylinder are widely known; therefore, we will not consider them here (see, for example, [9]). To take into account the influence of the windows in the cylinders, we can use the analysis of the resistance of the intercar spaces of a moving train presented in [10], since this problem is completely analogous to the problem considered. In [10], a formula for determining the friction stress over the generating lines of a train in intercar portions is given:

$$\tau = 0.019\,\rho\,\frac{u^2}{2},$$

where u is the velocity of the train. The resistance force of the windows on one side of the rotating cylinder will be

$$P = 2\pi R\,\tau ba/(a + l),$$

while the torque moment will be

$$M = PR.$$

The rotating central cylinder is similar, owing to the existence of windows in it, to the impeller of centrifugal pumps. The unidirectional radial flow rate is due to the action of the centrifugal force arising as a result of the rotation of the cylinder, and the radial flow in apparatuses of this type should be analyzed with the use of the Euler theory of centrifugal pumps. According to this theory, the energy equation for the motion of an ideal liquid through an opening in a rotating cylinder along the generating line of the opening is

$$\frac{p_{2\text{int}}}{\rho g} + \frac{v_{2\text{int}}^2}{2g} = \frac{p_{1\text{ext}}}{\rho g} + \frac{v_{1\text{ext}}^2}{2g} - \frac{\omega^2\,(r_{1\text{ext}}^2 - r_{2\text{int}}^2)}{2g}. \tag{1}$$

Here, the velocity along the generating line of the opening is denoted by v to decrease the number of indices in the notation. The indices in the notation coincide with the indices at r according to Fig. 1 and correspond to the values of the parameters at these distances from the center of the apparatus. In the case where the circular and radial velocities are orthogonal, the heads of the liquid at the inlet of the cylinder and at the outlet from it will be

$$H_{2\text{int}} = \frac{p_{2\text{int}}}{\rho g} + \frac{v_{2\text{int}}^2 + \omega^2 r_{2\text{int}}^2}{2g}, \quad H_{1\text{ext}} = \frac{p_{1\text{ext}}}{\rho g} + \frac{v_{1\text{ext}}^2 + \omega^2 r_{1\text{ext}}^2}{2g}. \tag{2}$$

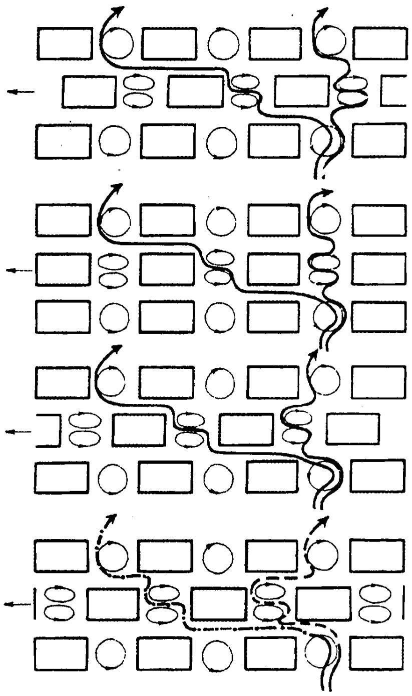
The theoretical head H_t developed by the rotating cylinder is equal to the difference of the heads at the exit from the cylinder window and at the entry to the window:

$$H_\text{t} = H_{1\text{ext}} - H_{2\text{int}} = \frac{p_{1\text{ext}} - p_{2\text{int}}}{\rho g} + \frac{v_{1\text{ext}}^2 - v_{2\text{int}}^2}{2g} + \frac{\omega^2\,(r_{1\text{ext}}^2 - r_{2\text{int}}^2)}{2g}. \tag{3}$$

Having substitutial the expression for the pressure difference into (3), in accordance with (1) we obtain

$$H_\text{t} = \frac{\omega^2\,(r_{1\text{ext}}^2 - r_{2\text{int}}^2)}{g}. \tag{4}$$

Since formula (4) has been derived for the case of rotation of an ideal liquid, i.e., a liquid free of internal friction, the value of H_t corresponds most closely to the conditions of the absence of the transit flow rate ($Q = 0$) through the



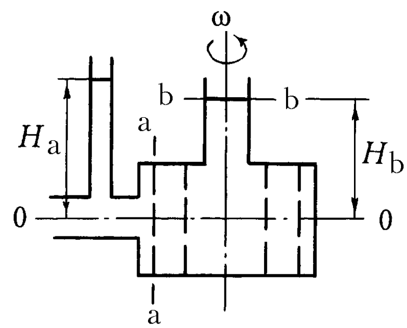


Fig. 4. Kinematics of compacts in an RPA.

Fig. 5. Calculation scheme of an RPA.

RPA. However, the real head H_0 will be smaller than the theoretical one also at $Q = 0$ because of the energy loss by the circulation flow within the RPA according to the scheme of Fig. 3:

$$H_0 = \eta_0 H_t , \tag{5}$$

where η_0 is the hydraulic efficiency of the apparatus at $Q = 0$. In the case $Q > 0$, the efficiency of the apparatus decreases: $\eta < \eta_0$.

The volumetric output of the RPA corresponds to the rate of liquid flow through the windows of the rotating cylinder

$$Q = 2\pi r_{1ext} b \frac{a}{a+l} v_{1ext} , \tag{6}$$

while the useful energy loss by the external movement of the liquid is determined as

$$N = \rho g H Q , \tag{7}$$

where $H = H(Q)$.

To find v_{1ext} and determine the head–flow-rate characteristics of the RPA, we use the Bernoulli equation for a uniformly rotating flow channel as applied to the calculation scheme of the RPA shown in Fig. 5. The cross sections a–a and b–b are taken as the calculation cross sections, and the plane with mark 0–0 is taken as the comparison surface:

$$H_a + \frac{v_{1ext}^2}{2g} = H_b + \frac{v_b^2}{2g} + H_0 - \Delta h , \tag{8}$$

where Δh is the energy loss due to the radial movement of the transit flow. The kinetic head in the cross section b–b can be neglected. In deriving the Bernoulli equation, allowance must be made for the fact that the circular motion of the liquid in the intercylinder gaps is initiated by the rotation of the central cylinder set in motion by the torque moment applied from the outside. Because of this, in (8) there is no component determining the kinetic energy of the transit circular flow. As has already been mentioned, the total transit flow is subdivided into two branches — circular and radial ones that then join together at the outlet from the immovable exterior cylinder (Fig. 4). As a result of the movement of particles in the RPA, the circular transit flow necessarily changes to a radial one, and, as has been mentioned above, the radial transit flow is possible in the RPA only in the form of compact tubes of flow. The energy loss by the change of the direction of the transit circular flow to the radial one Δh_{φ} can be compensated for only with the work of the centrifugal forces determined by the quantity H_0 or with the differential head in the radial direction. Since thereupon both branches of the total flow join together, the value of Δh_{φ} must be equal to the energy loss in the radial branch Δh_r:

$$\Delta h_{\varphi} = \Delta h_r = \Delta h \,. \tag{9}$$

The quantities Δh_{φ} and Δh_r qualify as local resistances; therefore, they are represented as

$$\Delta h_{\varphi} = \zeta_{\varphi\varepsilon} \frac{v_{\varphi\varepsilon}^2}{2g} \,, \quad \Delta h_r = \zeta_{r\varepsilon} \frac{v_{r\varepsilon}^2}{2g} \,. \tag{10}$$

Here $v_{\varphi\varepsilon}$ and $v_{r\varepsilon}$ are the velocities in the corresponding compact tubes of flow, and $\zeta_{\varphi\varepsilon}$ and $\zeta_{r\varepsilon}$ are the total coefficients of local resistances. The velocities $v_{\varphi\varepsilon}$ and $v_{r\varepsilon}$ exceed significantly the mean velocities calculated by relating the flow rates Q_{φ} and Q_r to the total area of the flow section. The value of $v_{\varphi\varepsilon}$ in a "compact" is determined by the value of $\omega r_{1\text{ext}}$:

$$v_{\varphi\varepsilon} = k_v \omega r_{1\text{ext}} \,, \tag{11}$$

where $k_v < 1$. The kinetic energy of the "compact" of the transit circular flow is in proportion to the relation between the transit and total parts of the circular flow rate:

$$q_{\varphi} = \frac{Q_{\varphi}}{Q_{\omega}} = \frac{2Q_{\varphi}}{\omega r_{1\text{ext}} \delta b} \,. \tag{12}$$

With account for (11) and (12) the first relation of (10) can be written as

$$\Delta h_{\varphi} = k_{\varphi} \frac{2Q_{\varphi}}{\omega r_{1\text{ext}} \delta b} \frac{k_v^2 \omega^2 r_{1\text{ext}}^2}{2g} \,, \tag{13}$$

or in the form

$$\Delta h_{\varphi} = \frac{k_{\varphi}}{\dfrac{v_{\varphi}}{k_v^2 \omega r_{1\text{ext}}} \dfrac{\delta}{2am}} \frac{Q_{\varphi}^2}{2gS^2} \,, \tag{14}$$

where

$$v_{\varphi} = \frac{Q_{\varphi}}{S} \tag{15}$$

is the velocity of the transit circular flow, averaged over the entire flow section

$$S = 2\pi r_{1\text{ext}} b \frac{a}{a + l} = abm \tag{16}$$

In such an event, the quantity

$$\varepsilon_{\varphi} = \sqrt{\frac{v_{\varphi}}{k_v^2 \omega r_{1\text{ext}}} \frac{\delta}{2am}} \tag{17}$$

should be interpreted as the effective contraction coefficient of the transit circular flow, and k_{φ} will correspond to the total coefficient of local resistances:

$$k_{\varphi} = \zeta_{\varphi\varepsilon} \,. \tag{18}$$

Since $k_v^2 << 1$ and $v_{\varphi} << \omega r_{1\text{ext}}$, it may be assumed to a first approximation that

$$\frac{v_{\varphi}}{k_v^2 \omega r_{1\text{ext}}} \approx 1 \,. \tag{19}$$

Let us introduce the notation

$$\zeta_{\varphi} = \frac{\zeta_{\varphi\varepsilon}}{\varepsilon_{\varphi}^2} \,. \tag{20}$$

Then (14) takes the form

$$\Delta h_{\varphi} = \zeta_{\varphi} \frac{Q_{\varphi}^2}{2gS} \,. \tag{21}$$

According to the diagrams of the main flows presented in Fig. 4, the number of rotations of the transit circular flow by the angle $\pi/2$ is equal to 8. Assuming that the value of $\zeta_{\varphi\varepsilon}^{(1)}$ of one rotation by the angle $\pi/2$ is equal to 1.2 [11], according to (17)–(21), we obtain

$$\zeta_{\varphi} = \frac{9.6}{\dfrac{\delta}{2am}} \,. \tag{22}$$

For the radial transit flow, in addition to the distinct 12 rotations by the angle $\pi/2$, it is necessary to take into account the local resistances which are due to the overlapping of the flow section by the rotor. Because of this, the second formula of (10) will be represented in the form

$$\Delta h_r = \left(\frac{14.4 + 1}{\varepsilon_r^2} + \zeta_{\text{d}} \right) \frac{Q_r^2}{2gS^2} \,, \tag{23}$$

or

$$\Delta h_r = \zeta_r \frac{Q_r^2}{2gS^2} \,, \tag{24}$$

where ζ_{d} is the coefficient of resistance of the diaphragm that opens and closes periodically; ε_r is the effective coefficient of contraction of the radial transit flow.

Equation (23) takes into account the loss by the generation of the kinetic energy of a "compact" of the radial transit component of the flow. The coefficient ζ_{d} is usually related to the kinetic energy of the flow prior to the contraction, which is reflected in dependence (23). Since the values of ζ_{d} change from zero to infinity with time, the flow rate Q_r must be variable. In this case, an equation of the type (8) should be supplemented with an inertial component and be written, with account for (9), as follows:

$$H_b + H_0 - \left(\frac{15.4}{\varepsilon_r^2} + \zeta_d(t)\right)\frac{v_r^2}{2g} = H_a + \frac{X}{g}\frac{dv_r}{dt}, \tag{25}$$

where

$$v_r = \frac{Q_r}{S}, \tag{26}$$

and X is the length of the inertial portion. For apparatuses of the type presented in Fig. 1, $X \approx 3L$. Analogously to (25), we compose the equation for calculating the head H_d before the rotating cylinder:

$$H_b - \frac{1 + 5 \cdot 1.2}{\varepsilon_r^2}\frac{v_r^2}{2g} = H_d + \frac{X_d}{g}\frac{dv_r}{dt}. \tag{27}$$

According to the diagram of transit flows in Fig. 4, Eq. (27) takes into account five rotations of the radial "compact" by the angle $\pi/2$ before the entry into the window of the rotor. Here, $X_d \approx L$. The head H_0 is produced by the rotating cylinder and, because of this, is not included in Eq. (27). For the time scale, it is convenient to take

$$\tau = \frac{a + l}{\omega r_{1\text{ext}}} \tag{28}$$

and to use the dimensionless quantities

$$\bar{t} = \frac{t}{\tau}, \quad \bar{S} = \frac{S(\bar{t})}{S}. \tag{29}$$

in calculations. The latter quantity in (29) characterizes the degree of opening of the windows relative to the immovable cylinders. The dependence of ζ_d on $\bar{S}$ was approximated by the data presented in [12]:

$$\bar{S} < 0.005 \qquad \zeta_d = \frac{1.153}{0.005^{2.30}},$$

$$\bar{S} = 0.005 - 0.300 \qquad \zeta_d = \frac{1.153}{\bar{S}^{2.30}},$$

$$\bar{S} = 0.30 - 0.80 \qquad \zeta_d = 10^{(-3.28\bar{S} + 2.244)},$$

$$\bar{S} = 0.80 - 1.00 \qquad \zeta_d = -4.33 \log \bar{S}. \tag{30}$$

When the Cauchy problems were solved, the initial conditions ($\bar{t}$ = 0) adopted for Eqs. (25) and (27) corresponded to the total overlapping of the windows of the immovable cylinders by the wall of the rotor ($\bar{S}$ = 0). In the interval $\bar{t} \in [0, 1.0]$, the dependences of $\bar{S}$ on $\bar{t}$ are as follows:

$$\bar{S} = 2\bar{t} \quad \text{at} \quad \bar{t} \in [0, 0.5], \quad \bar{S} = 2(1 - \bar{t}) \quad \text{at} \quad \bar{t} \in [0, 1.0].$$

The value of ε_r must be of the same order of magnitude as ε_φ. A numerical solution of the Cauchy problems has shown that even at small X ($X \sim 10^{-2}$ m) and very large heads

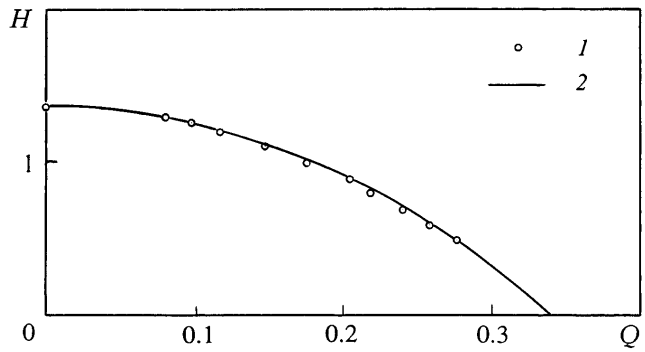


Fig. 6. Head–flow-rate characteristic of RPA-1. H, m; Q, liters/sec.

$$h = H_0 - (H_a - H_b) \tag{31}$$

($h \sim 10$ m) the distributions of the velocities and rarefactions ($H_d - H_b$) in the intervals $\bar{t} \in [0.05, 0.98]$ are practically uniform. Because of this, the calculation can be made from the "truncated" Eq. (25) (without the last term on the right-hand side of (25)). According to these calculations, the fluctuations of the flow rate Q_r can also be neglected and the problem of determination of Q_r can be solved as a pseudostationary problem.

Having determined the structure of the expressions for Δh_φ and Δh_r, we return to the initial equation (3) which, with account for (31), will be written in the form

$$h = \frac{Q^2}{2gS^2} + \Delta h\,, \tag{32}$$

where the energy loss Δh is expressed in terms of the total coefficient of resistance ζ:

$$\Delta h = \zeta \frac{Q^2}{2gS^2}\,. \tag{33}$$

Then the head–flow-rate characteristic of the RPA will be

$$Q = 4.43S \sqrt{\frac{H_0 - H}{1 + \zeta}}\,, \tag{34}$$

where $H = H_a - H_b$. The value of ζ and the correspondence of (34) to the real head–flow-rate characteristics are determined experimentally. By way of example, Fig. 6 compares dependence (34) to experimental data for an RPA-1, whose parameters are presented below in Table 1. As follows from these data, the predicted square law of resistances in the RPA is confirmed.

The next problem is derivation of expressions to determine the subdivision of the transit flow Q into the components Q_r and Q_φ as well as expressions to calculate ε_r. For its solution we have the following system of relations:

$$1)\ \Delta h = \zeta \frac{Q^2}{2gS^2}\,,$$

$$2)\ \Delta h = \zeta_\varphi \frac{Q_\varphi^2}{2gS^2}\,,$$

TABLE 1. Parameters of Rotory-Pulsatory Apparatuses

Parameter	Values		Form of characteristics
	RPA-1	RPA-2	
$r_{1\mathrm{ext}}$, mm	33.53	33.85	Design
$r_{2\mathrm{b}}$, mm	30.18	30.10	
δ, mm	0.50	0.17	
a, mm	2.78		
l, mm	2.78		
L, mm	3.53	3.75	
b, mm	22.40		
ω, rad/sec	298		
m	36		
S, dm^2	0.2358	0.2381	Calculation by design data
τ, msec	0.557	0.552	
$Q_{\omega}^{(1)}$, liters/sec	0.056	0.019	
$Q_{\varphi,\max}$, liters/sec	2.014	0.691	
H_{t}, m	1.932	2.170	Experimental
H_0, m	1.37	1.38	
Q_0, liters/sec	0.340	0.354	
η_0	0.709	0.636	Solution of equations
ζ	1292	1224	
ζ_{φ}	3843	11300	
ζ_r	7321	2718	
ε_{φ}	0.0500	0.0291	
ε_r	0.0467	0.0771	
$\langle \zeta_{\mathrm{d}} \rangle$	150.1	96.6	
ζ_{di}	2206		
q_{φ}	0.580	0.329	
q_r	0.420	0.671	
$\bar{v}_{\varphi}$	11.60	11.29	
$\bar{v}_r$	9.00	8.70	
$\bar{h}_{\varphi,\max}$	73.95	22.38	

$$3)\ \Delta h = \left(\frac{\zeta_{r\varepsilon}}{\varepsilon_r^2} + \langle \zeta_{\mathrm{d}} \rangle \right) \frac{Q_r^2}{2gS^2} = \langle \zeta_r \rangle \frac{Q_r^2}{2gS^2} ,$$

$$4)\ \Delta h = \left(\frac{\zeta_{r\varepsilon}}{\varepsilon_r^2} + \zeta_{\mathrm{d}} (\bar{t}) \right) \frac{Q_r^2 (\bar{t})}{2gS^2} = \langle \zeta_r \rangle \frac{Q_r^2}{2gS^2} , \tag{35}$$

$$5)\ Q = Q_r + Q_{\varphi} .$$

The middle component of the third relation in (35) is written in a form corresponding to the quasistationary conditions. Because of this, the quantity $\zeta_{\mathrm{d}}(t)$ is replaced here by a certain averaged parameter $\langle \zeta_{\mathrm{d}} \rangle$. The middle component of the fourth relation in (35) reflects the nonstationary character of the radial transit flow. Since the quantity Δh does not change with time, the changes in $\zeta_d(\bar{t})$ and $Q_r^2(\bar{t})$ must be such that the last component of the fourth relation in (35) is constant. Thus, the equality

$$\frac{Q_r^2(\bar{t})}{Q_r^2} = \frac{\langle \zeta_r \rangle}{\dfrac{\zeta_{r\varepsilon}}{\varepsilon_r^2} + \zeta_d(\bar{t})} . \tag{36}$$

is observed. The quantity $Q_r^2(\bar{t})$ must satisfy the mean-integral equivalence condition Q_r^2:

$$Q_r^2 = \int_0^1 Q_r^2(\bar{t})\, d\bar{t} , \tag{37}$$

which leads to the integral relation

$$\int_0^1 \frac{\langle \zeta_r \rangle}{\dfrac{\zeta_{r\varepsilon}}{\varepsilon_r^2} + \zeta_d(\bar{t})} d\bar{t} = 1 , \tag{38}$$

which allows one to find the value of ε_r if the function $\zeta_d(\bar{t})$ is prescribed. Relation (38) is solved by the iteration method.

When system (35) is solved, it is assumed that the quantity ζ_φ is known from (22) and the quantity ζ is known from the experimental data. Then, according to the first, second, and fifth equations in (35), the total transit flow in the RPA is subdivided into the circular and radial components

$$q_\varphi = \frac{Q_\varphi}{Q} = \sqrt{\frac{\zeta}{\zeta_\varphi}} , \quad q_r = \frac{Q_r}{Q} = 1 - q_\varphi , \tag{39}$$

and the first and third relations in (35) allow one to calculate the quantity

$$\langle \zeta_r \rangle = \frac{\zeta}{q_r^2} . \tag{40}$$

Substituting (40) into (38), we find the value of ε_r and then the value of $\langle \zeta_d \rangle$ from the third relation in (35):

$$\langle \zeta_d \rangle = \langle \zeta_r \rangle - \frac{\zeta_{r\varepsilon}}{\varepsilon_r^2} . \tag{41}$$

The dynamics of changes of $Q_r(\bar{t})$ in the interval $\bar{t} \in [0, 1.0]$ is determined by the expression

$$\frac{Q_r(\bar{t})}{Q_r} = \sqrt{\frac{\langle \zeta_r \rangle}{\dfrac{\zeta_\varepsilon}{\varepsilon_r^2} + \zeta_d(\bar{t})}} , \tag{42}$$

while the mean velocities in the "compacts" are determined by the formulas

$$v_{\varphi\varepsilon} = \frac{Q_\varphi}{S\varepsilon_\varphi}, \quad v_{r\varepsilon} = \frac{Q_r}{S\varepsilon_r}; \quad \bar{v}_\varphi = \frac{q_\varphi}{\varepsilon_\varphi}, \quad \bar{v}_r = \frac{q_r}{\varepsilon_r}. \tag{43}$$

Equations (32) and (33) allow one to determine the limiting head $h_{\max}$, at which the pressure gradient in the intercylinder gaps remains zero in the azimuth direction:

$$h_{\max} = \frac{1+\zeta}{2g}\left(\frac{Q_{\varphi,\max}}{q_\varphi S}\right)^2, \tag{44}$$

where

$$Q_{\varphi,\max} = \frac{\omega r_{1\text{ext}}\delta b}{2} m. \tag{45}$$

The quantity $h_{\max}$ is conveniently expressed in dimensionless form:

$$\bar{h}_{\max} = \frac{h_{\max}}{H_{\mathrm{t}}}. \tag{46}$$

As an example we present (Table 1) a summary of design, calculation, and experimental characteristics for two modifications of a concrete RPA differing in the intercylinder gaps δ. In the table, we also present parameters calculated according to the above-described calculation method. An increase in δ caused the corresponding change in the circular flow rate $Q_\omega^{(1)}$ per intercylinder gap and an increase in the maximum possible transit flow rate $Q_{\varphi,\max}$ at which the azimuthal pressure gradient in the gap still remains equal to zero. When $Q_{\varphi,\max}$ is compared to the maximum value of the resultant transit flow rate Q_0, detected in the experiments, it is apparent that a "gradient-free" flow occurs in the gaps of the RPA. The hydraulic efficiency η_0 is fairly high at $Q = 0$. The total coefficients of resistance ζ of RPA-1 and RPA-2 were found to be close. However, the distributions of the transit flows in the modifications are significantly different. A threefold increase in ζ_φ in RPA-2 caused a decrease of approximately $\sqrt{\zeta_\varphi}$ in the circular transit flow in it. The smaller size of the gap δ provided the greater compactness of the circular tube of flow ($\varepsilon_{\varphi1} < \varepsilon_{\varphi2}$). At the same time, the mean values of the velocities in the compacts characterized by the parameters $\bar{v}_\varphi$ and $\bar{v}_r$ were found to be close. The significant difference between $\langle\zeta_{\mathrm{d}}\rangle$ and its mean-integral value ζ_{di} is noteworthy, which underlines the necessity of using Eq. (38), which determines the value of $\langle\zeta_{\mathrm{d}}\rangle$ along with ε_r. Comparison of q_φ of the RPA modifications allows the conclusion that RPA-1 is more preferable from the standpoint of increasing the portion of the medium flowing through the intercylinder gaps.

Thus, the method proposed allows one to determine the hydrodynamics of the internal processes in an RPA and calculate the parameter q_φ, which is a diagnostic variable in terms of technology.

NOTATION

a, width of the window, m; *b*, height of the cylinders, m; *k*, coefficient; *H*, head, m; Δh, head loss, m; *Q*, volumetric flow rate, m^3/sec; *q*, relation between the flow rates; *g*, free-fall acceleration, m/sec^2; *L*, thickness of the cylindrical shells, m; *l*, interslot distance, m; *m*, number of slots in the cylinders; *M*, torque moment, N·m; *N*, power, W; *p*, pressure, Pa; *r*, radius, m; *R*, central radius of the rotating cylinder, m; *t*, time, sec; *u*, local velocity, m/sec; *v*, mean velocity, m/sec; δ, width of the intercylinder gap, m; μ, dynamic coefficient of viscosity, Pa·sec; ν, kinematic coefficient of viscosity, m^2/sec; η, hydraulic efficiency; φ, angle in the cylindrical coordinate system, rad; φ_i, velocity coefficient; ζ, coefficient of hydraulic resistances; ω, angular velocity, rad/sec; ρ, density, kg/m^3; τ, friction stress, Pa. Subscripts and superscripts: *r* and φ, component of the quantity in the direction *r* and φ; 1, smaller radius of the gap; 2, larger radius of the gap; 1 and 2, as the second indices denote the parameters of RPA-1 or RPA-2, respectively; a and b, in the cross sections a and b; bound, at the boundary; *v*, velocity; ext, external; int, internal; 0, at the center, at zero value; d, baffle (diaphragm); ε, with allowance for the contraction of the flow; ω, under the action of rotation of the rotor; i, integral; *, critical values. t, theoretical; $\langle\ \rangle$, symbol of averaging; max, maximum.

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